ST MICHAEL'S
Church of England High School

Calculation Policy

Progression in Calculation

St Michael's Church of England High School Mathematics Department

Addition	My addition stage is…	
Subtraction	My subtraction stage is…	
Multiplication	My multiplication stage is…	
Division	My division stage is…	

OUR SCHOOL PRAYER:

What does the Lord ask of you?
To act justly, to love mercy and to walk humbly with your God.
Micah 6vs.8

ST MICHAEL'S
Church of England High School

MERCY • INTEGRITY • CARE AND COMPASSION • HUMILITY • ACHIEVEMENT • EQUALITY • LEADERSHIP • SERVICE

This document provides an overview of strategies used to teach calculations. These methods will continue to be used in all mathematics classrooms to support the learning of pupils and need to be remembered and employed by all staff when doing calculations.

Laid out below are the written methods pupils should use when performing basic arithmetic; staff should use them when demonstrating such. These strategies show a progression for each of the four operations. Pupils may come into your classroom with differing levels of mathematical ability and therefore could be using the full variety of methods. Each teacher needs to make sure that they are allowing each pupil to use the method they feel confident with so that we are not confusing them but are instead furthering their mathematical development. Teaching a student a set of rules for a method whose concept they cannot grasp will not help them progress; it will often cause misconceptions which take time to undo. If in doubt revert to the simplest method available or check with a member of the mathematics team in advance.

It is important that children do not abandon core mathematical skills used at KS2 and KS3. Therefore, pupils will always be encouraged to look at a calculation/problem and then decide which choice of method is best to use. They will do this by asking themselves:

‘Can I do this in my head?’
‘Can I do this using drawings or jottings?’
‘Do I need to use a pencil and paper procedure?’
‘Do I need a calculator?’

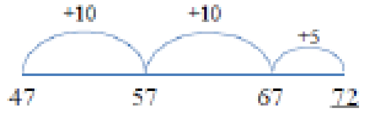
A useful website for mathematical definitions is “amathdictionaryforkids.com”. This is an American website and should be used with caution.

Addition Progression

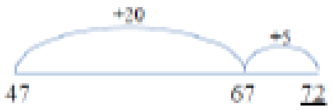
Stage 1

Use a number line to make jottings.

e.g. 47 and 25



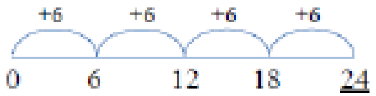
or



Drawing an empty number line helps record the steps taken in a calculation. Break down the number you want to add on and then add it on in steps. The steps can be broken down as much or as little as required.

As a follow through step this method is useful when times tables are not known

e.g. 4 × 6



However it is restrictive in its lack of efficiency for larger number therefore the times tables up 10 × 10 should be learnt as soon as possible

Stage 2

Use partitioning to reflect mental methods.

e.g. 487 + 546

$$\begin{array}{rrr} 400 & 80 & 7 \\ +\,500 & 40 & 6 \\ \hline 900 & 120 & 13 \end{array} = 1033$$

Partition the sum into tens and units. Then combine answers. This method is potentially faster than the number line and can easily be extended to hundreds and beyond.

Stage 3

Expanded column method

e.g. 487 + 546

$$\begin{array}{rr} & 546 \\ + & 487 \\ \hline & 900 \\ & 120 \\ & 13 \\ \hline & 1033 \end{array}$$

To ensure understanding the significance (value) of each digit should be emphasized. The calculations are 500 + 400 (not 5+4), then 40 + 80 (not 4+8), then 6 + 7 and finally 900 + 120 +13. At this stage you should still be dealing with the most significant part of the calculation first.

Stage 4

Column method (decomposition)

e.g. 487 + 546

$$\begin{array}{r} 546 \\ +\,487 \\ \hline {}^{1}\,{}^{1} \\ 1033 \\ \hline \end{array}$$

Add the digits in the columns, starting from the right hand side. To ensure accuracy note any numbers you 'carry over'.

Stage 5

Column addition with decimals.

e.g. 54.6 + 48.7

$$\begin{array}{r} 54.6 \\ +\,48.7 \\ \hline {}^{1}\,{}^{1} \\ 103.3 \\ \hline \end{array}$$

Same as column method, add the digits, starting from the right hand side. Make sure to 'carry over' to the next column as normal.

e.g. 54.6 + 4.87

$$\begin{array}{r} 54.6\color{red}{0} \\ +\;\;4.87 \\ \hline {}^{1} \\ 59.47 \\ \hline \end{array}$$

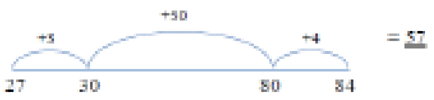
Take care when placing the digits in the correct columns, you may wish to include a zero (0) as a place holder.

Subtraction Progression

Stage 1

Number line (count up)

e.g. 84 – 27



Count up from the smaller to the larger number in manageable steps; these are then added together to find the total difference. This method develops understanding of the link between subtraction and difference.

Stage 2

Number line (count back)

e.g. 834 – 328

Break down the number you want to subtract and then take it off in steps. The steps can be broken down as much or as little as required. This method is rarely the most efficient, but it is a stepping stone to partitioning.

Stage 3

Partitioning

e.g. 834 – 328

$$
\begin{array}{rrrl}
 & 20 & 14 & \\
800 & \cancel{30} & \cancel{4} & \\
-\ 300 & 20 & 8 & \\
\hline
500 & 0 & 6 & = 506
\end{array}
$$

Partition the sum into hundreds, tens and units. Subtract, starting from the units. Where it is not possible to subtract (e.g. 40 – 70) borrow from the next column. This method is an introducing to decomposition.

Stage 4

Column method (decomposition)

e.g. 834 – 328

$$
\begin{array}{rccc}
 & 8 & \overset{2}{\cancel{3}} & {}^{1}\cancel{4} \\
- & 3 & 2 & 8 \\
\hline
 & 5 & 0 & 6
\end{array}
$$

Subtract the digits in columns, starting from the right hand side. To ensure accuracy note any changes from numbers you 'borrow'. This method is the same as the partitioning method except place value is used to denote the significance of digits.

Stage 5

Column method with decimals.

e.g. 59.8 – 48.7

$$
\begin{array}{rcccc}
 & 5 & 9 & . & 8 \\
- & 4 & 8 & . & 7 \\
\hline
 & 1 & 1 & . & 1
\end{array}
$$

e.g. 58.6 – 4.87

$$
\begin{array}{rccccc}
 & 5 & {}^{7}\cancel{8} & . & {}^{1}{}^{5}\cancel{6} & {}^{1}0 \\
+ & & 4 & . & 8 & 7 \\
\hline
 & 5 & 3 & . & 7 & 3
\end{array}
$$

Same as column method, subtract the digits, starting from the right hand side. Make sure to 'borrow' from the next column as normal. Take care when placing the digits in the correct columns, you may wish to include a zero (0) as a place holder.

Multiplication Progression

Stage 1	Stage 2	Stage 3	Stage 4	Stage 5
Partitioning	**Grid Method**	**Expanded Column Method (Partitioning method)**	**Column Method (short multiplication)**	**Column Method (Long multiplication)**
e.g. 36 × 8 36 x 8 (30 + 6) x 8 240 + 48 = £288 or 10 x 8 = 80 10 x 8 = 80 10 x 8 = 80 6 x 8 = 48 36 x 8 = £288	e.g. 72 × 34 x | 70 | 2 30 | 2100 | 60 = 2160 4 | 280 | 8 = 288 2448 cm	e.g. 24 x 7 24 X 7 28 140 168 e.g. 72 x 34 72 X 34 8 280 60 2100 2448 1	e.g. 24 x 7 24 X 7 168 2 Answer: 168	Eg. 72 x 34 72 X 34 288 2160 2448 1 Answer: 2448
Break the larger numbers into smaller ones that you can work with. Working must be clearly laid out so it is obvious how this has been done.	The grid method is a formal method of partitioning. It is relatively efficient for large calculations but unlike more advanced methods still displays the significance of each number (100 is shown as 100 not 1) so mistakes or misconceptions are less likely.	Column method requires students to have an understanding of place value (for example 20 x 7 = 140 not 14). Misconceptions are more likely, however by encouraging students to clearly set out the digits in the correct place value column misconceptions can be reduced.	Short multiplication requires students to have a solid understanding of long multiplication, to ensure they are placing the digits in the correct place value column. Students should be encouraged to set out the digits neatly to avoid any misconceptions.	Once students have grasped the concept of place value and mastered short multiplication, they should be able to answer questions where they are required to multiply both two digit and three digit numbers.

Further Guidance: for current pupils, it is integral for the students to be working towards the column method. To avoid building misconceptions that may occur at this stage, each teacher needs to make sure that they are allowing each pupil to use the method they feel confident with so that we are not consuming them but are instead furthering their mathematical development. If in doubt, revert of a simper method available or check with a member of the mathematics team in advance.

Division

Progression

Stage 1

Sharing

e.g. 48 ÷ 4

10, 1, 1	10, 1, 1
10, 1, 1	10, 1, 1

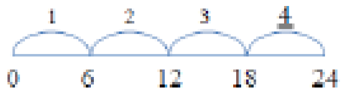
$10 + 1 + 1 = \underline{12}$

Draw 4 boxes and share parts of 48 into them. Depending on ability this may be tallied one at a time, or more quickly.

Stage 2

Number line (counting on)

e.g. 24 ÷ 6



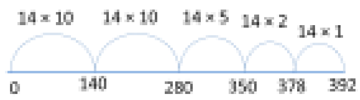
Count up in steps until you reach the target number. This method is useful when times tables are not known. It is restrictive in its lack of efficiency for larger numbers therefore the times tables up to 10 × 10 should be learnt as soon as possible.

Stage 3

Chunking

e.g.

396 ÷ 14 =



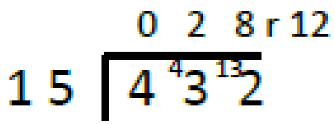
396 ÷ 14 = 28 r4

Count up in chunks until you reach the target number. The chunks can be as large or as small as required. As understanding develops move away from using number line.

Stage 4

Bus Stop method (Short division)

e.g. 432 ÷ 15



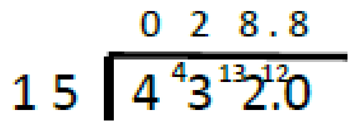
Answer: $28\frac{12}{15}$

The divisor should be placed outside of the "bus stop". Calculate how many times the divisor fits into the far left digit. Write this above the "bus stop" and carry any remainders down. Repeat for the remaining digits.

Stage 5

Bus stop method

e.g. 432 ÷ 15



Answer: 28.8

The divisor should be placed outside of the "bus stop". Calculate how many times the divisor fits into the far left digit. Write this above the "bus stop" and carry any remainders across to the following digit. Repeat for the remaining digits. Add the decimal point and a zero to carry across the remainder.

Further Guidance

For whole school numeracy we do not teach beyond the bus stop method. This is because any further efficiency gained is either not required nor worth the misconceptions that regularly occur beyond this level. However, where individual students are successfully using their own method they should be left alone to continue.

Remainders

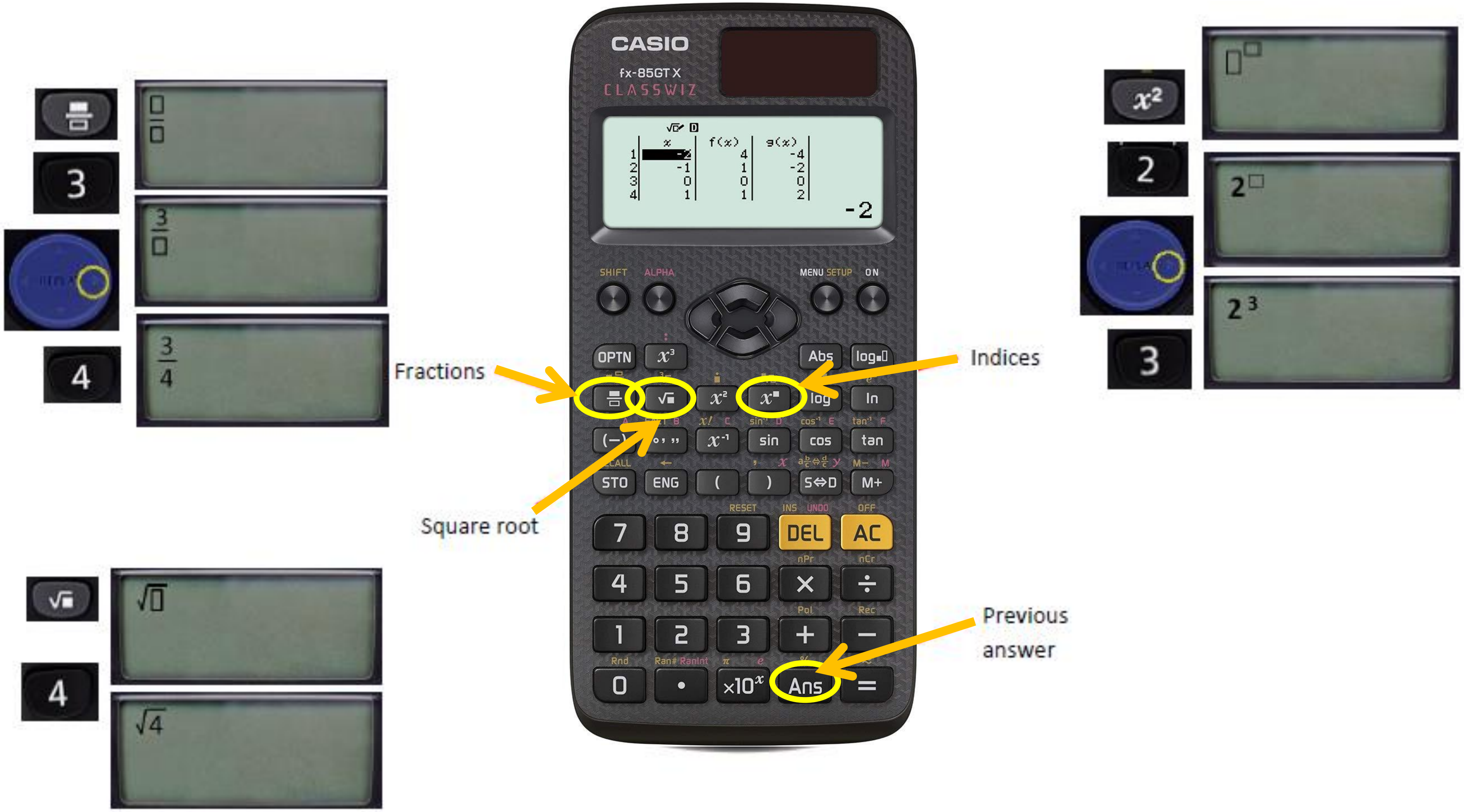
All the examples above have no remainder. When questions have a remainder, students should write them as such. They should not the write the remainder as a decimal until they have a complete understanding that these are not the same thing. The table below shows guidance on progression her:

Question	Solution	Progression	Decimal Solution	Misconception
30 ÷ 8	3 r6	$3\,{}^{6}/_{8}$	3.75	3.6

Use of calculator

Only use calculators where calculations are complex. Pupils should be encouraged to use written methods across all subjects to reinforce what is taught in mathematics.

It is important to make sure that pupils write down the calculation they have put in the calculator.



Appendix 1: Examples of formal written methods for addition, subtraction, multiplication and division

This appendix sets out some examples of formal written methods for all four operations to illustrate the range of methods that could be taught, it is not intended to be an exhaustive list, nor is it intended to show progression in formal written methods. For example, the exact position of intermediate calculations (superscript and subscript digits) will vary depending on the method and format used.

For multiplication, some pupils may include an addition symbol when adding partial products. For division, some pupils may include a subtraction symbol when subtracting multiples of the divisor.

Addition and subtraction

789 + 642 becomes

	7	8	9
+	6	4	2
1	4	3	1
	1	1	

Answer: 1431

874 – 532 becomes

	8	7	4
-	5	2	3
	3	5	1

Answer: 351

932 – 457 becomes

	8	12	1
	$\cancel{9}$	$\cancel{3}$	2
-	4	5	7
	4	7	5

Answer: 475

932 – 457 becomes

		1	1
	9	3	2
-	$\cancel{4}$	$\cancel{5}$	7
	5	6	
	4	7	5

Answer: 475

Short multiplication

24 x 6 becomes

	2	4
x		6
1	4	4

Answer: 1431

342 x 7 becomes

	3	4	2
x			7
2	3	9	4
	2	1	

Answer: 2394

2741 x 6 becomes

	2	7	4	1
x				6
1	6	4	4	6
	4	2		

Answer: 16,446

Long multiplication

24 x 16 becomes

	2	
	2	4
x	1	6
2	4	0
1	4	4
3	8	4

Answer: 384

124 x 26 becomes

	1	2	
	1	2	4
x		2	6
2	4	8	0
	7	4	4
3	2	2	4
1	1		

Answer: 3224

124 x 2.6 becomes

	1	2	
	1	2	4
x		2	6
	7	4	4
2	4	8	0
3	2	2	4
1	1		

Answer: 322.4

Short division

98 ÷ 7 becomes

	1	4
7	9	^{2}8

Answer: 14

432 ÷ 5 becomes

		8	6	r 2
5	4	3	^{3}2	

Answer: 86 remainder 2

496 ÷ 11 becomes

			4	5	r 1
1	1	4	9	^{5}6	

Answer: 45 $\frac{1}{11}$

Long division

432 ÷ 15 becomes

		2	8	r 12
1	5	4	3	2
		3	0	0
		1	3	2
		1	2	0
			1	2

Answer: 28 remainder 12

432 ÷ 15 becomes

			8	6	
1	5	4	3	2	
		3	0	0	15x20
		1	3	2	
		1	2	0	15x8
			1	2	

$\frac{\cancel{12}}{\cancel{15}} = \frac{4}{5}$

Answer: 28 $\frac{4}{5}$

432 ÷ 15 becomes

			2	8	.	8
1	5	4	3	2	.	0
		3	0			
		1	3	2		
		1	2	0		
			1	2		0
			1	2		0
						0

Answer: 28.8